Mathematical Modeling of the Human Cognitive System in Two Serial Processing Stages with its Applications in Determining Optimal Interval of In-vehicle Messages in Adaptive Workload Management Systems

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Abstract

With an increasing usage of in-vehicle systems, drivers have to frequently perceive and respond to the messages from these in-vehicle systems. And previous studies have found that interval between the messages (arrival rate) presented to a driver becomes one of factors affecting driver workload. To reduce driver workload, researchers in adaptive workload management system (Wu et al., 2008) have found that adding extra delay time into the interval of messages can significantly reduce driver workload. However, it is unknown whether this extra delay time added by an adaptive workload management system will increase the performance time of drivers or not. To answer this important question, using closed-form mathematical equations, the current work quantifies human performance time (total task completion time (TTC) and reaction time of each task) when there are two serial processing stages in the human cognitive system. The mathematical model developed in this work provides solutions of the optimal interval of messages that generate lowest workload without deteriorating drivers' performance time to respond multiple messages from in-vehicle systems. This is one of few closed-form deterministic mathematical models with analytic solutions which can predict average reaction time when there are two multiple serial stages in the cognitive system in dual tasks. With relatively simple equations, the mathematical model can still capture the major patterns of simulation results with stochastic properties and human behavioral experimental results. The mathematical equations developed in this study can be used in the designing of adaptive workload management system as well as other driver assistance systems.

Keywords: Driver Workload, Adaptive System, Queueing Cognitive Model, Serial Processing

1. Introduction

With the development of technology, there is increased use of many vehicle information systems (e.g., road and travel guidance, directions, sensors and detectors, and vehicle status) [1-3], vehicle safety/warning systems (e.g., lane departure, collision, and curve speed warnings), and vehicle communication systems (e.g., vehicle-to-vehicle communication and use of cellular phones while driving). Multitasking between driving and using these systems may impose high information load on drivers, increasing their mental workload [4-6], which in turn increases the chance of vehicle collisions compared to a single driving condition [4, 7]. This introduces a very important topic in in-vehicle system design and transportation safety—how to present information from these in-vehicle systems properly to improve driver performance and reduce driver workload.

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One of the most important factors in affecting driver performance and workload is the interval between tasks or messages [8-10], which refers to the temporal delay from the arrival of a task or message presented to a driver to the arrival of the next task or message. For example, when a driver is operating a vehicle, he or she receives a message from an in-vehicle device (e.g., a road guidance system) and needs to respond to this message; at the same time or after a while (the length of this duration is the interval between messages), the driver gets another message from the same or another in-vehicle system (e.g., a cellar phone) for another manual or vocal response. In psychology and human factors, this interval between the arrivals of messages/tasks is also called Stimulus Onset Asynchrony (SOA) [8]. In designing intelligent transportation systems, an optimal interval or SOA between these messages or tasks from in-vehicle devices means that an interval generates lowest workload without deteriorating drivers' performance time in completing multiple tasks from in-vehicle systems.

The interval between tasks or messages is composed of two components: the first component is the actual interval between messages or tasks determined by when these multiple messages/tasks actually happen in the real world; the second component is additional interval added by an intelligent transportation system [10]. In the following part of this introduction, the effect of the interval on driver workload and performance is described in detail.

From the perspective of driver workload, if tasks and messages are presented to a driver very close to each other, due to the limited information capacity of human operator, driver workload typically increases significantly compared to the condition when intervals between messages or tasks are relatively long. Wu et al. [10] conducted an experimental study which found that both young and elder drivers perceived higher mental workload compared when the interval between two messages decreases (reducing the length of the second component in the interval). Based on the computational modeling of human performance and mental workload using Queueing Network-Model Human Processor (QN-MHP) (Wu et al., 2004-2008) [8, 11] , Wu et al. (2007) also found that a decrease of intervals between messages (represented by an increase of interval arrival rate λ) increases the predicted workload level (under the condition that the other variables in the equations remain the same value) [19][1].

However, from the perspective of driver performance, whether prolonging/extending the interval between presentations of messages/tasks (by prolonging the second component in the interval) will affect the driver performance becomes an important research question. Intuitively thinking, any additional time or extra waiting time of messages added into a human-machine system might prolong the human-machine system's response time. However, without build quantitative models or/and performing empirical studies, it is very hard to test this intuitive hypothesis is true or not. To explore this question in detail, the following concepts need to be introduced.

1) Total task completion time (TTC): total task completion time refers the duration between the time point when the first stimuli was presented and time point when the response of the last stimuli was made by a human operator. If there are two messages or tasks, the total task completion time is the duration between the time point when the stimuli of the first message/task was presented and time point when the response of the

[1] However, this predicted decrease of workload only means the value of workload decreases, it dose not mean the decrease has to reach significant $p<.05$ level in statistics [5].

second message/task was made. Compared to reaction time of individual tasks, total task completion time (TTC) provides a more complete index of the whole human-machine system in multitasking situations, since it includes one more variable in multitasking—the interval between the presentation times of stimuli (also called stimulus onset asynchrony, SOA) [8, 9].

2) Stimulus onset asynchrony (SOA) and adaptive workload systems: Wu et al. [10] developed an adaptive workload system for drivers, which can control (either reduce or prolong) the extra delay time (the second component in SOA or interval between messages as described above) to reduce driver workload based on different driving difficulty, properties of the in-vehicle tasks, and characteristics of drivers. Wu et al's research (2007) has found that the additional interval added by an adaptive workload can significantly reduce driver workload; however, it is unknown whether the additional interval added by an intelligent transportation system may increase TTC or not. If TTC increases by additional interval, this kind of adaptive system may not be suitable when TTC is the major concern in transportation safety. For example, if there are two warning signals (one from pedestrian warning system and the other from the front collision system) and if it is found that the additional interval added by the adaptive systems will increase TTC of a driver, then these adaptive systems may not be recommended to manage the message intervals in these urgent situations. However, if it is found that TTC does not increase (or at least keeps the same) even additional interval is added, these adaptive systems may be used when TTC is the major concern in transportation safety.

3) Serial and parallel processing: in order to quantify the relation between SOA and TTC, it is necessary to identify the serial and parallel information processing in the human cognitive system. Each information processing stage of the human cognitive system (e.g., perceptual, cognitive and motor execution) can process information in a serial (processing stimuli of one task one by one) or a parallel manner (processing stimuli of multiple tasks at the same time) [20, 21]. For example, people can perceive digital numbers on a visual display one by one (serial processing in the perceptual stage); people can also perceive visual and auditory information at the same time (parallel processing in the perceptual stage). Accordingly, there are four possible situations in the cognitive system: 1, 2, 3 and more than 3 serial stages in the cognitive system [22][2]. To predict TTC, mathematical equations have been developed in the paradigm of psychological refractory period (PRP) when there is only one (single) serial stage in the cognitive system (please see the description in the following PRP section). Therefore, this paper focuses on the prediction of TTC when there are 2 serial stages in the cognitive system and conducting corresponding experiments to validate this prediction which has not been developed by existing studies in human factors or psychology.

In practice, many in-vehicle systems' tasks involve 2 serial stages in the cognitive system. For example, a driver perceives an auditory messages and another message on an in-vehicle display or perceives two visual stimuli on an in-vehicle display at the same time (i.e., parallel information processing in the perceptual stage), then the driver makes a decision related to each message (i.e., serial in the cognitive stage) and executes his or her decision using his right hand (e.g., press the first and second button on the in-vehicle

[2] Since most of tasks involve cognitive stage, especially decision making, and it is found that it is extremely hard that this cognitive stage is able to work in a parallel manner [26], currently 0 serial stage in the cognitive system is not considered.

display, serial processing in motor stage). If drivers can perceive the messages in a serial manner (e.g., perceive two messages from an auditory display, or perceive two visual stimuli one by one on an in-vehicle display), make decisions respectively, and execute their responses using right foot and one of their hands at the same time, it is also 2 serial stages (serial in perceptual and cognitive stage) involved in the cognitive system. Moreover, in theory, 2-serial stage is the simplest case of multiple serial stages in the cognitive system. Once the 2-serial stage situation is modeled, it can be used as a platform to quantify the behavior of the cognitive system with 3 or more serial stages.

4) Psychological refractory period (PRP): PRP is the basic experimental paradigm to study multitasking performance. Typically, subjects in PRP are asked to perform two choice reaction tasks at the same time: subjects perceive stimuli of the two tasks (one from the visual channel and the other from auditory channel), and make responses of the two tasks via two hands or feet separately, indicating there is one serial stage (cognitive stage) in the human cognitive system. The relation between SOA and reaction time is summarized in Figure 1 and corresponding equations can be found in work of Pashler [23-25], and similar relation was also found in driving context [26].

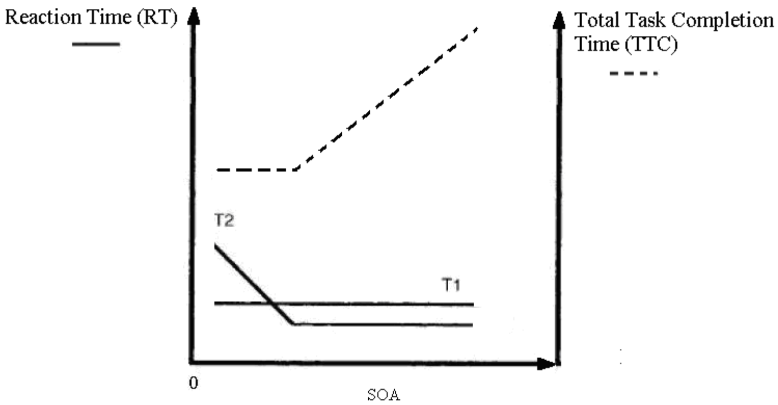

Figure 1. The relation between SOA, reaction time and total task completion time (TTC) in single serial stage condition (T1: Task 1; T2: Task 2)

In Figure 1, when SOA is short, TTC keeps constant because of a decrease of reaction time of Task 2. In other words, when the interval between the two tasks/messages is short and there is only one (single) serial processor in the cognitive system (e.g., a driver perceives two message in parallel in the perceptual stage and outputs the motor action using different body parts (one with hand the other with foot)), additional interval can be added (i.e., the second component of the interval can be positive) without affecting the driver's TTC in performing the tasks.

However, whether the TTC still keeps constant in short SOA condition when there are two serial stages in human cognitive system, is still unknown since corresponding equations and mathematical model have not been developed by existing research. To answer this important question in multitasking, this paper introduces a set of mathematical equations and also describes the corresponding experiments to validate the prediction of these mathematical models.

2. Modeling Human Performance Time in the Situation of 2 Serial Processing Stages

In the following section of this paper, a set of mathematical equations is proposed to quantify human performance time including the Total Task Completion Time (TTC), Reaction Time of T1 (RT1) and Reaction Time of T2 (RT2), when there are 2 serial stages in the human cognitive system.

1) Definition:

Ai: Duration starting from time 0 (T1 presents) to the time point when the task *i* entity enters the first serial stage (BF).

BFi: Processing time of task *i* entity at the first serial stage.

Ci: Sum of processing time of task *i* entity after it leaves the first serial stage but before it enters the second serial stage (BS)

BSi: Processing time of task *i* entity at the second serial stage (BS)

Di: Processing time of task *i* entity after it leaves BS

Bottleneck of Entities at Stage i: When entities of a task have to wait for the service of a stage who is fully occupied by other entities, bottleneck of entities at Stage *i* occurs. A serial stage does not necessarily produce the bottleneck of entities at that stage.

Total Task Completion Time (TTC): Duration starting from time 0 (T1 presents) to the time point when the response of T1 or T2 is made. $TTC=max(RT1,RT2+SOA)$. TTC is the major index of human performance since RT1 or RT2 does not include SOA (SOA is the additional delay of T2 which may deteriorate human performance).

To calculate TTC and reaction time of T1 and T2, there are two serial stages in the cognitive system is analyzed in two different conditions: Condition 1: bottleneck of entities occurs at the first serial stage (BF); Condition 2: no bottleneck of entities occurs at the first serial stage (BF).

2) Development of Mathematical Models

<u>Condition 1. Bottleneck of entities occurs at the first serial stage</u> (Mathematical Expression: $A1-BF2<A2\leq A1+BF1$)

When entities of T1 or T2 have to wait for the service of the first serial stage (BF) which is occupied by the other entities, bottleneck of entities at BF occurs. Equation 1 quantified this situation which includes two major conditions: 1) T2 arrives at BF earlier than T1 ($A1-BF2 <A2\leq A1$) and 2) T1 arrives at BF earlier than T2 ($A1 <A2 \leq A1+BF1$). The following part of this section describes these two major conditions in detail.

$$A1-BF2 < A2 \leq A1+BF1 \quad (1)$$

<u>Condition 1.1: T2 arrives at BF earlier than T1</u> (Mathematical Expression: $A1-BF2<A2\leq A1$)

Depending on the situation at the second serial stage (BS), Condition 1.1 is composed of three sub-conditions:

Condition 1.1.1 T2 arrives at BS earlier than T1 (Bottleneck of entities at BS: first condition)

This condition can be represented by following Equation 2:

$$A2+BF2+BF1+C1-BS2\leq A2+BF2+C2<A2+BF2+BF1+C1 \quad (2)$$

TTC and reaction time of the two tasks can be quantified by following equations:

$$RT1=A2+BF2+C2+BS2+BS1+D1+\Gamma \quad (3)$$

where Γ represents the additional processing time of a primary task (e.g., driving of a experienced driver). The value of Γ changes depending on the difficulty level of that primary task.

$$RT2=A2+BF2+C2+BS2+D2-SOA+\Gamma \quad (4)$$
$$TTC=max(RT1,RT2+SOA) \quad (5)$$

Condition 1.1.2 T1 arrives at BS earlier than T2 (Bottleneck of entities at BS: second condition)

This condition can be represented by following Equation 6:

$$A2+BF2+BF1+C1\leq A2+BF2+C2\leq A2+BF1+BF2+C1+BS1 \quad (6)$$

TTC and reaction time of the two tasks can be quantified by following equations:

$$RT1=A2+BF2+BF1+C1+BS1+D1+\Gamma \quad (7)$$
$$RT2=A2+BF2+BF1+C1+BS1+BS2+D2-SOA+\Gamma \quad (8)$$
$$TTC=max(RT1,RT2+SOA) \quad (9)$$

Condition 1.1.3 No bottleneck of entities at BS

In the condition 1.1.3 ($A2+BF2+C2<A2+BF1+BF2+C1-BS2$ *OR* $A2+BF2+C2> A2+BF1+BF2+C1+BS1$), TTC and reaction time of the two tasks can be quantified by following equations:

$$RT1=A2+BF2+BF1+C1+BS1+D1+\Gamma \quad (10)$$
$$RT2=A2+BF2+C2+BS2+D2-SOA+\Gamma \quad (11)$$
$$TTC=max(RT1,RT2+SOA) \quad (12)$$

<u>Condition 1.2: T1 arrives at BF earlier than T2</u> (Mathematical Expression: $A1<A2<A1+F1$)

Similar to condition 1.1, the condition 1.2 (entities of T1 arrive at the first serial stage (BF) earlier than those of T2) also includes three sub-conditions:

Condition 1.2.1 T2 arrives at BS earlier than T1 (Bottleneck of entities at BS: first condition) ($A1+BF1+BF2+C2<A1+BF1+C1\leq A1+BF1+BF2+C2+BS2$)

$$RT1=A1+BF2+BF1+C2+BS1+BS2+D1+\Gamma \quad (13)$$
$$RT2=A1+BF1+BF2+C2+BS2+D2-SOA+\Gamma \quad (14)$$
$$TTC=max(RT1,RT2+SOA) \quad (15)$$

Condition 1.2.2 T1 arrives at BS earlier than T2 (Bottleneck of entities at BS: second condition)

($A1+BF1+BF2+C2-BS1\leq A1+BF1+C1\leq A1+BF1+BF2+C2$)

$$RT1=A1+BF1+C1+BS1+D1+\Gamma \quad (16)$$
$$RT2=A1+BF1+C1+BS1+BS2+D2-SOA+\Gamma \quad (17)$$
$$TTC=max(RT1,RT2+SOA) \quad (18)$$

Condition 1.2.3 No bottleneck of entities at BS

($A1+BF1+C1<A1+BF1+BF2+C2-BS1$ *OR* $A1+BF1+C1> A1+BF1+BF2+C2+BS2$)

$$RT1=A1+BF1+C1+BS1+D1+\Gamma \quad (19)$$
$$RT2=A1+BF1+BF2+C2+BS2+D2-SOA+\Gamma \quad (20)$$
$$TTC=max(RT1,RT2+SOA) \quad (21)$$

<u>2. No Bottleneck of entities occurs at the first serial stage (BF)</u>

($A2\geq A1+BF1$ *OR* $A2\leq A1-BF2$)

2.1 T2 arrives at the second stage (BS) earlier than T1 (Bottleneck of entities at BS: first condition) ($A2+BF2+C2<A1+BF1+C1\leq A2+BF2+C2+BS2$)

$$RT1=A2+BF2+C2+BS1+BS2+D1+\Gamma \quad (22)$$

$$RT2=A2+BF2+C2+BS2+D2-SOA+\Gamma \quad (23)$$

$$TTC=max(RT1,RT2+SOA) \quad (24)$$

2.2 T1 arrives at the second stage (BS) earlier than T2 (Bottleneck of entities at BS: second condition) ($A2+BF2+C2-BS2<A1+BF1+C1\leq A2+BF2+C2$)

$$RT1=A1+BF1+C1+BS1+D1+\Gamma \quad (25)$$

$$RT2= A1+BF1+C1+BS1+BS2+D2-SOA+\Gamma \quad (26)$$

$$TTC=max(RT1,RT2+SOA) \quad (27)$$

2.3 No bottleneck at the second stage (BS) ($A1+BF1+C1<A2+BF2+C2-BS2$ *OR* $A1+BF1+C1> A2+BF2+C2+BS2$)

$$RT1=A1+BF1+C1+BS1+D1+\Gamma \quad (28)$$

$$RT2= A2+BF2+C2+BBS2+D2-SOA+\Gamma \quad (29)$$

$$TTC=max(RT1,RT2+SOA) \quad (30)$$

In other words, the value of *TTC* is depending on the value of parameters, including processing time of the two types of entities before, on and after each serial stage. More importantly, using the mathematical equations developed above, designers of intelligent transportation system including different in-vehicle warning and message systems, will predict the length of TTC in the 2 serial stages condition and determine whether it is appropriate to add additional time between the messages/information of two tasks: if *TTC* kept constant in certain SOA based on the model, it means the additional time added between the messages of two tasks may not prolong drivers' total performance time while drivers' mental workload decreased with the additional delay between messages and information.

In the following sections, we describe a case study with potential practical importance to validate the prediction of the mathematical model including the value of SOA when *TTC* starts to change and predicted value of *TTC* in different SOA conditions.

3. A Case Study with Practical Importance

According to a report from NHTSA's National Center for Statistics and Analysis, speeding is one of the most prevalent factors contributing to traffic crashes: The economic cost to society of speeding-related crashes is estimated by NHTSA to be $40.4 billion per year; in 2004, speeding was a contributing factor in 30 percent of all fatal crashes, and 13,192 lives were lost in speeding-related crashes [27, 28]. Traffic law enforcement (police officers detecting speeding and issuing speeding tickets) is one of the most critical measures to prevent speeding. However, aside from detecting speeding, police officers also have to perform other tasks at the same time, e.g., communicating with dispatchers and navigating the vehicle to a target location. The following experimental paradigm in multitasking in driving was introduced by Wu et al. (2007, 2008) [10, 29] based interviews with police officers.

Speeding detection or judgment task (Radar_Vis Task) (Subtask 1): Officers need to read two numbers on a display of an in-vehicle radar system mounted on the dashboards of police vehicles. The first number is the speed of a target vehicle measured by the radar system; the second is the distance from the police vehicle to the target vehicle. Whether the target vehicle is speeding is determined by both the speed and the distance. For example: a) If the speed is within the range from 55 to 60 yards (including 55 and 60), an officer need to see the distance, if the distance is beyond 65 yards (including 65), then it is moderate speeding (level II)→ press “II” button; if the distance is below 65 yards, it is severe speeding (level I) → press “I” button. b) If the speed is above 61 yards (including 61), an officer need to see the distance, if the distance is beyond 105 yards (including 105), then it is moderate speeding (level II)→ press “II” button; if the distance is below 105 yards, it is severe speeding (level I) → press “I” button.

Radio message response task (Mesg_Aud Task) (Subtask 2): Auditory messages received by the officers usually come from multiple sources (headquarters, other police officers, and maintenance), and the officers need to respond to higher priority messages (i.e., messages from headquarters) by pressing a button on the radio.

The most frequent order of these two tasks, based on the interview [10, 29], is a radar speeding detection task followed by a message response task (the duration between presentation of the numbers in the speed detection and the presentation of the voice message of the message response task is the interval between messages or SOA in this paper)[3,4]. This sample multitasking scenario of police officers was also inspired by the ALERT project at the Texas Transportation Institute, which focused on the development of an integrated interface of various devices (radar detection system, radio, video recording systems, etc.) for police officers to improve their performance and safety [30].

This sample multiple-task can also be generalized into other multitasking situations in driving since it captures several important characteristics of multitasking in driving: 1) It considers one of the most important variables in multitasking—interval (delay time) between the presentation of information of different tasks; 2) Multitasking information in driving is typically presented in a multimodal format, either through the visual (e.g., looking at a map or a display of a navigation system) or the auditory modality (e.g., listening to messages from cellular phones or warning systems); 3) It covers perceptual, cognitive, and motor processing in multitasking. For example, the speed detection task might be similar to a secondary task in using a navigation system while driving: Drivers read directions for and the distance to the next turn from the display (perceptual processing), perform mental calculations to decide whether and when to switch to a different lane (cognitive processing), and possibly engage the turning signal and turn the steering wheel (motor processing).

In the example task, there are two serial stages in the information processing of the two tasks: cognitive stage (judgment and decision making stage) and motor execution stage—one of the hand processor (i.e., right hand for pressing the buttons of the message response task or the speeding judgment task), the corresponding terms can be specified into: $BFi=Fi$, $BSi=Handi$, $Di=0$. The expected value of these parameters are estimated based on the original value of these parameters in QN-MHP [31] and Fitts’ law (See estimation of these parameters in Appendix 1).

Figure 2 shows the predicted TTC pattern in this case study when SOA increases from 0 to 3 sec based on the mathematical model developed in Section 2. First, when SOA

[3] Since the sample task is composed of a pair of two subtasks: speeding detection task (*RTs*) followed by a message response task (*RTm*), the reaction time of the secondary task (*ST*) as a representative performance index of the whole secondary task is defined as: $ST=(RTs+RTm)/2$

[4] This message delay time in the majority of multitasking cases, based on the interview, is longer than 3 s.

increases from 1.5 to 2 sec, it is predicted that there is a relatively large increase (0.5 sec) of TTC. Second, from SOA=0 to 1 sec, TTC keeps constant or at least it will not increases significantly. Third, interestingly, the slope of TTC when SOA increases from 1.5 to 3 sec, is predicted to be greater than the slope of TTC when SOA increases from 1 to 1.5. This prediction of TTC is different from the situation when there is only one serial processing stage in the cognitive system (if there is only one serial stage, the increase of TTC will follow the same slope [32]). In addition, there is a small increase of TTC (0.21 sec) when SOA increases from 1 to 1.5 sec. Fourth, TTC at the longest SOA condition (SOA=3 sec) is much longer than TTC at all of the other SOA conditions. In addition, based on mathematical models built in the previous study [19], driver workload will decreases with an increase of SOA.

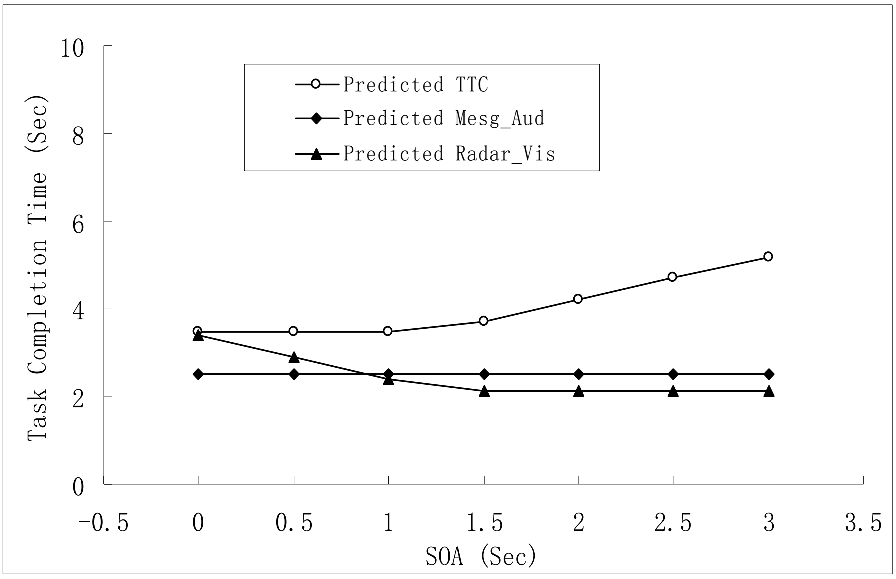

Figure 2. The predicted TTC based on the mathematical model

In other words, based on the mathematical model developed in this study and previous study, in order to reduce driver workload and at the same time maintain driver's performance in the Mesg_AUD and Radar_fVIS tasks at the initial level (SOA=0, no extra delay time is added), the optimal delay time is at SOA=1.5 under the condition that a relatively small (greater than 0 but less than 0.5 sec) increase of TTC is acceptable for an in-vehicle system (e.g., some non-urgent messages/tasks in the vehicle, including message-response task described in this case study). If an in-vehicle system cannot accept any increase of TTC (e.g., some warning messages requires drivers' immediate response), then the optimal delay time is at SOA=1 given the same sets of task parameters described in this case study.

In the following part of the paper, we described the verification of the mathematical model using both discrete-event simulation and a behavioral experiment.

Simulation Verification

A discrete event simulation of the 2 serial stage system was performed to verify the prediction of the mathematical model. A queue system with four servers was built (See Figure 3): the first and the third server had infinite capacity; the capacity of the second and fourth server is one (The processing times of these servers followed a triangular distribution (isosceles triangle) with its mode equals to the same value of the parameters in Appendix 1, min=0, and max=2*mode).

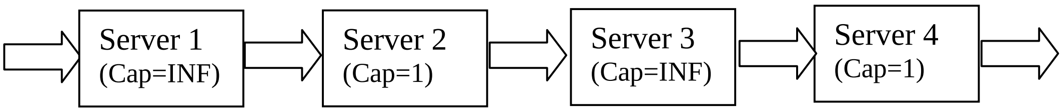

Figure 3. Discrete event simulation model of a four-server tandem queue to verify the prediction of the mathematical model (Cap: Capacity)

This four-server tandem queue was implemented in Promodel® as frequently-used discrete event simulation software. After running the simulation model for 120 replications, the simulation results of the simulation model and well as the predicted value from the mathematical model were plotted in Figure 4. Even though the average variance of the simulation results is relatively high (mean SD= 835 ms), the mathematical model can still capture the major pattern of the simulation results (R square=0.96, RMS=177 ms).

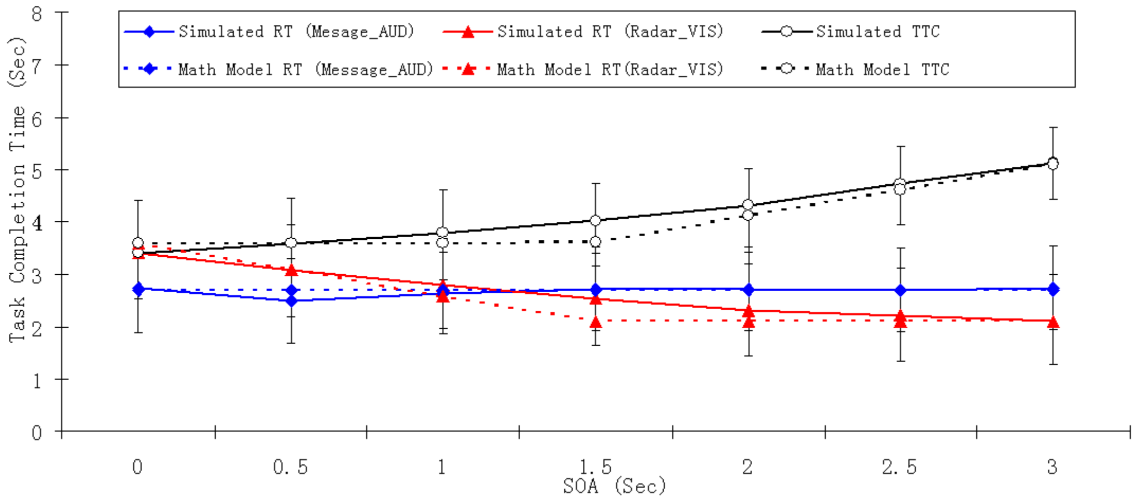

Figure 4. The simulation results compared with the prediction of the mathematical model (Error bar represents +/- 1SD of the simulation results)

Behavioral Experiment Verification

1) Participants

16 students (8 male and 8 female) from State University of New York (SUNY) at Buffalo participated this experiment (average age: 24 years old; SD of age: 3.4). All of the subjects are right-handed and had corrected far visual acuity of 20/40 or better and midrange (80 cm) visual acuity of 20/70 or better. On average, the subjects have 5.7 years' driving experience (SD=1.8) and they received payment as a compensation to participate this experiment. These participants were selected according to the mathematical model's current setting (experienced driver: see parameter Γ and estimation of parameters' value in Appendix).

2) Equipment

A STISIM® driving simulator (STISIMDRIVE M100K) was used in the experimental study. The STIMSIM simulator was installed on a Dell Workstation (Precision 490, Dual Core Intel Xeon Processor 5130 2GHz) with a 256MB PCIe x16 nVidia graphic card, Sound Blaster® X-Fi™ system, and Dell A225 Stereo System. The driving scenario was presented on a 27-inch LCD with 1920X1200 pixels resolution. The driving simulator also included a Logitech Momo® steering wheel with force feedback and a gas and a break pedal.

The control panel of the secondary task was simulated by a 12.1 inch ELO touch screen which was located at 50 cm from the right hand of the subjects and 91 cm from the eyes of subjects. The visual angle of the touch screen is 13.1 degree (The touch screen has approximately 100 ms response time when a finger presses on its screen). This touch screen was controlled by a Dell PC (OPTIPLEX 745) which was connected with the driving simulator via Labjack® system.

3) Experimental Procedure and Design

The current experiment was similar to the experimental paradigm in the study conducted by Wu et al. [10]. For the primary driving task, subjects are asked to drive on the right lane of a straight road with no traffic in the right lane. Subjects are also instructed to maintain their speed in 45 miles/hr. If they are driving 10 mi/hr above or below the speed shown on the speed-limit signs, each participant heard a computer-generated voice saying “too fast” or “too slow”.

Besides the primary task, there are two subtasks in the secondary tasks which were presented to the subjects: Mesg_Aud Task and Radar_Vis Task. Figure 5 shows the user interface (coded with Visual Basic Application (VBA) in Excel) of the multimodal system in this experiment. And it includes the two pairs of response keys for the radar judgment and message response task. This VBA program was installed on the Dell PC which controlled the LCD display, it also automatically recorded the response time and TTC of subjects in the experiment.

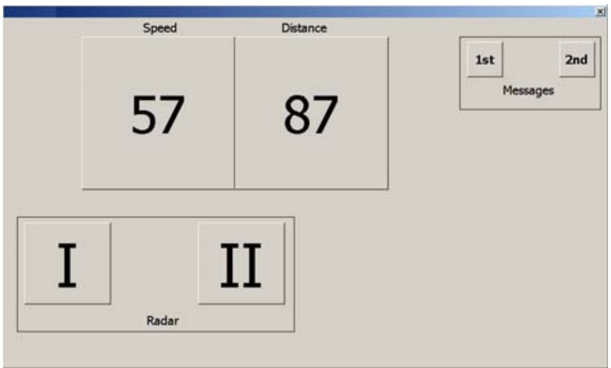

Figure 5. The user interface of the multimodal in-vehicle system

For the Mesg_Aud task, when subjects heard the word “first dispatches” (the presentation duration of the word “first” was 300 ms in the auditory modality) from the speakers, they were asked to double click on the “1st” button on the touch screen with their right index fingers; if they heard “second dispatches” (the presentation duration of the word “second” was the same with that of word “first”), they were instructed to double click on the “2nd” button on the touch screen with the same fingers (this double clicking was designed to mimic the flipping of switches on a physical dispatch panel which took 500 ms on average).

For the Radar_Vis task, subjects were asked to judge the level of speeding of another vehicle based on speed and distance information on the touch screen the using following rules (the presentation duration of the speed and distance information was 5 seconds in the visual modality): a) If the speed is within the range of 55 to 60 (including 55 and 60), they need to see the distance: If the distance is beyond 65 yards (including 65), they were asked to press the “II” button because it is moderate speeding (level II). If the distance is below 65 yards, they were instructed to press the “I” button since it is severe speeding (level I). b) If the speed is above 61 (including 61), they need to see the distance: If the distance is beyond 105 yards (including 105), it is moderate speeding (level II) and subjects were asked to press “II” button; if the distance is below 105 yards, subjects were instructed to press “I” button because it is severe speeding (level I).

One factor within-subject design is used in the behavioral experiment to validate the prediction of the model. The independent variable is the SOA between the stimuli of the two subtasks, which includes seven levels (SOA=0, 0.5, 1, 1.5, 2, 2.5 and 3 seconds). After filling in the pretest forms and taking vision tests, participants first practiced the single task situations of 1) driving without a secondary task, and 2) performing the secondary task while the simulator was in the parked condition. Then, participants practiced dual task situations of driving while performing a secondary task at the same time. During the actual test, each subject went through the seven blocks corresponding to the seven levels of SOA (e.g., block 1: SOA=2.5, bloc 2: SOA=0.5, …etc). The order of SOA levels followed the revised Latin Square design to balance the order effect [33]. Within each block, the time between pairs of the two subtasks is random with a range from 5 to 20 seconds. After each block, subjects were asked to complete the NASA-TLX form to report their subjective workload in that SOA condition.

4) Results

Based on the results of ANOVA for TTC, the main effect of SOA on TTC was significant ($F(6, 90)=23.34$, $p<.001$) (see Figure 6). Table 1 shows the four homogenous subsets of TTC determined by Student-Newman-Keuls (SNK) procedure at the seven different SOA levels. The first subset includes SOA ranges from 0.5 to 1, which indicates that TTC at SOA=.5 and SOA=1 were significantly lower than TTC at the other SOA levels ($p<.05$, see Table 1). When SOA is at 0, 1 and 1.5 (Subset 2), TTC increases significantly compared to TTC in Subset 1. The third subset includes SOA=2 and SOA=2.5, which indicates that TTC at SOA=2 and SOA=2.5 are significantly higher than TTC at previous subset including SOA=0, SOA=1, and SOA=1.5. The last subset is SOA=3 in which TTC is higher than that in all of the other SOA conditions.

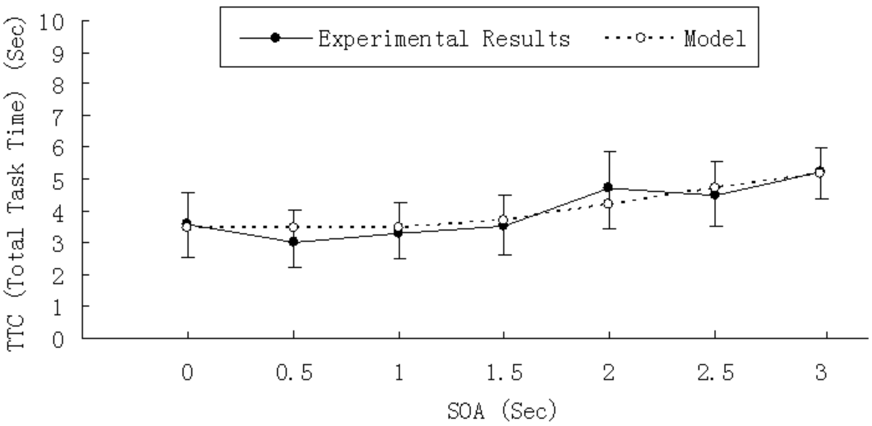

Figure 6. Experimental results of TTC (solid line) compared to the model's prediction (dashed line) (Error bar represents +/- 1SD of the data)

Table 1. Homogenous Subsets (underlined) of TTC at the seven SOA levels

	Homogeneous Subsets						
Subset 1	SOA=0.5	SOA=1					
	2.99	3.29					
Subset 2		SOA=1	SOA=1.5	SOA=0			
		3.29	3.54	3.60			
Subset 3					SOA=2.5	SOA=2	
					4.49	4.71	
Subset 4							SOA=3
							5.21

* Determined by Student-Newman-Keuls procedure with significance at $p<.05$ (The order of SOA levels is set based on the mean value of TTC)

* Format of this table is obtained from [34]

The major patterns of TTC in the experimental result of TTC are consistent with the prediction of the model. The R square of the model is .93 with RMS=.30 sec. First, when SOA is longer than 1.5 sec in the current task setting, there is a significant increase of TTC compared to TTC when SOA is less than 1.5 sec. In other words, the mathematical model developed in this paper is able to predict the optimal delay time (i.e., 1.5 second in this case study) of the secondary task presented to a human operator (e.g., a driver). As long as the sum of the additional delay time and the actual interval between the two messages is less than the predicted turning point (i.e., 1.5 second in this case study), the TTC may not increase significantly. Second, from SOA=0 to SOA=1, the mathematical model predicts that TTC keeps constant or at least does not increase during this period; the experimental results also confirmed there is no significant increase of TTC when SOA increases from 0 to 1. Third, the slope of TTC when SOA increase from 1 to 1.5 sec, is smaller than the slope of TTC when SOA increase from 1.5 to 3 sec (see Figure 6). In addition, there is a small increase of TTC (0.24 sec) when SOA increase from 1 to 1.5 sec. These experimental results are also consistent with model's prediction of TTC in this SOA range. Fourth, the model predicted that TTC keeps increasing when SOA increases and TTC at longest SOA will be larger than that in other SOA conditions; the experimental finding also confirmed that TTC at SOA=3 (Subset 4) is significantly longer than TTC at all of the other SOA conditions ($p<.05$, see Table 1).

Figures 7 and 8 show the comparison of reaction time between the model's prediction and experimental results in the Mesg_Aud and Radar_Vis Task. The R square of the comparison of reaction time predicted by the mathematical model and experimental results in Radar_Vis Task is .68 with RMS=.30 sec; The RMS of the comparison of reaction time predicted by the mathematical model and experimental results in Mesg_Aud Task is .36 sec[5]. These results also indicated the mathematical model can account for the main pattern of reaction time of the two tasks in different SOA levels.

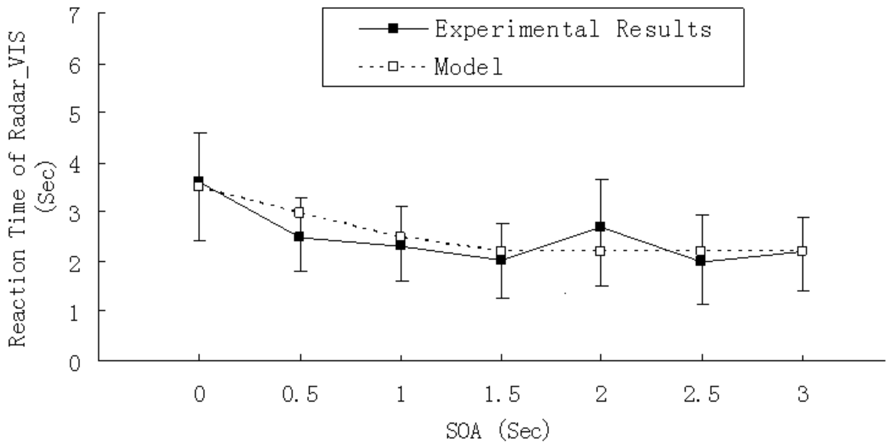

Figure 7. Comparison of reaction time predicted by the mathematical model and experimental results in Radar_Vis Task (Error bar represents +/- 1SD of the data)

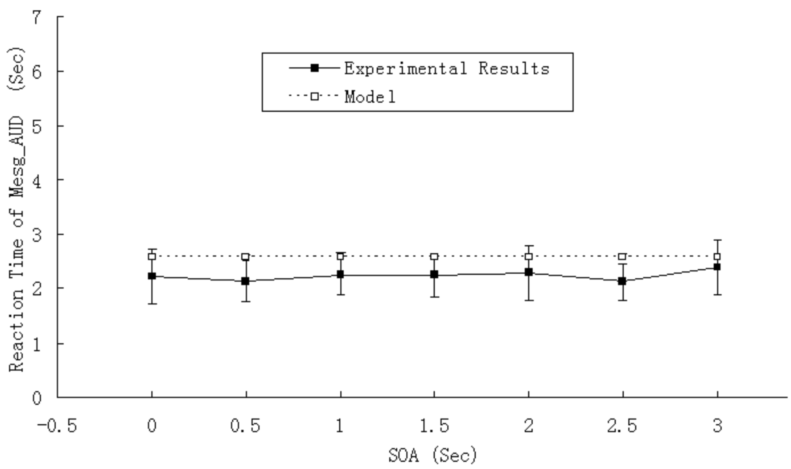

Figure 8. Comparison of reaction time predicted by the mathematical model and experimental results in Mesg_Aud Task (Error bar represents +/- 1SD of the data)

The effect of SOA on overall workload is shown in Figure 9. Driver workload keeps decreasing when SOA increases from 0 to 3. And ANOVA was performed and found the main effect of SOA on the overall workload is significant ($F(6, 90)=9.05$, $p<.01$).

[5] Since the model's prediction keeps constant in different SOA levels ((*x-x bar)*=0), the denominator of the correlation equation ($Correl(X,Y)=\sum(x-\bar{x})(y-\bar{y})/\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}$) becomes 0. Therefore, The R square of comparison of reaction time predicted by the mathematical model and experimental results in Mesg_Aud Task is not obtained.

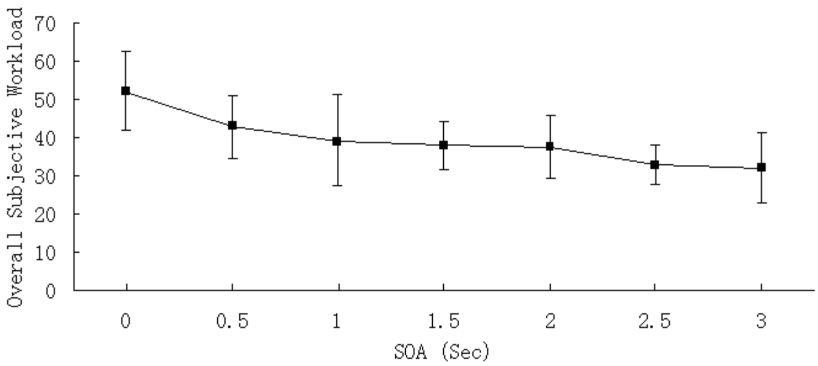

Figure 9. The change of overall workload at the seven levels of SOA
(Error bar represents +/- 1SD of the data)

In addition, ANOVA was performed and found the main effect of SOA on driving performance (standard deviation of deviation from central line) is not significant ($F(6, 90)=.53$, $p>.05$). The driving performance on the seven levels of SOA belongs to the same homogeneous subset based on the SNK procedure. The main effect of SOA on the error rates of the two secondary task (Mesg_Aud and Radar_Vis) is also not significant (5% error rate, $F(6, 90)=1.46$, $p>.05$ for Mesag_Aud Task; $F(6, 90)=.91$, $p>.05$ for Radar_Vis Task).

In sum, the experimental results of both driver performance (in driving task, the Mesg_Aud and Radar_Vis tasks) and workload, confirmed the model's prediction in the following points: 1) The experimental results in this case study indicated that the total task competition time (TTC) will increase its value significantly when SOA increases from 1.5 sec to 2 sec, which is consistent with model's prediction; 2) Based on the mathematical model of driver workload in the previous study in this line of research [19], the driver workload in the experiment decreases from SOA=0 to SOA=3. Thus, the optimal interval between the two messages is at 1.5 sec in this case study generating lowest driver workload without deteriorating driver performance in the in-vehicle tasks or driving performance.

Discussion

This work developed a set of mathematical model quantifying the human performance time (reaction time of tasks and total task completion time) when there are two serial processing stages involved in the human cognitive system. This is one of few closed-form deterministic mathematical models with analytic solutions which can predict average reaction time when there is multiple serial stages in the cognitive system in dual tasks under a driving context. With relatively simple equations, the mathematical model can still capture the major patterns of simulation results with stochastic properties and human behavioral experimental results. The major cognitive architectures (e.g., ACT-R and EPIC) can predict human performance in dual tasks; however, to predict the average reaction time, they have to rely on computer simulation and all of these models only modeled the mean of reaction time rather than the variations [8, 22]. In cognitive modeling and other modeling studies, there is a parsimony principle: A set of simple math equations which can capture the major characteristics of the system to be modeled, is usually regarded as better than lots extremely complex equations or sets of computer simulation codes [35]. The mathematical model is validated by both a discrete-event simulation model and a human behavioral experiment in driving. Specifically, the mathematical model can be used in the some real-world situations: For example, drivers perceive task information one by one, make corresponding decisions in a serial manner, and then execute motor response in a parallel manner with a foot and a hand together or with mouth and a hand together. Or drivers perceive task information in a parallel manner (both in auditory and visual modalities), make decisions of the tasks one by one and execute motor responses in a serial manner.

One of the major applications of this mathematical model is its usage in adaptive workload management systems for drivers. This mathematical model can be used in the algorithms of adaptive workload management systems to determine whether additional time can be added into the presentation of messages to a driver. The prediction of TTC and reaction time can be changed when there are different tasks or human driver involved in the human-machine system. The prediction of the model can inform the designer whether extra delay time can be added into the interval between different messages and what is the optimal extra delay time is, depending on the tolerance in prolonging of the total task completion time. As described in the case study, if an in-vehicle system (e.g., important safety warning systems) requires immediate response from drivers (low tolerance of in prolonging of TTC), the mathematical model gives a relatively smaller optimal delay time compared to in-vehicle systems which does not need immediate responses from drivers.

Moreover, the mathematical model can be easily implemented in the vehicle design software and the software of in-vehicle systems and predict human performance time in real-time. Given the tasks' properties, human information processing time of these tasks, the model can be directly coded in these systems. During real-world driving task, the model can at least provide a real-time estimation of human performance in the 2-serial stage situation, so that the in-vehicle system can adaptively change the messages to drivers or activate automatic response systems. For example, in certain multitasking situation, there is only 2 seconds left before the two vehicles collide to each other while mathematical model predicts that drivers may need 3 seconds to respond these two warning messages tasks. In this situation, an automatic system can be activated to stop the vehicle automatically to reduce the change of collisions.

There are several limitations for the current mathematical model. First, it only models the expected/average human performance time in the 2-serial stages without covering the distribution of human performance time in that situation. To model the distributions of human performance time, a multi-class tandem queuing model is to be developed to account for both expected human performance time and its variability. Second, sometimes, there are three or even more than three serial stages involved in the human cognition system. For example, drivers perceive messages one by one, make serial decisions, and execute motor movement in a serial manner. In this case, the current model needs be extended and more complex mathematical models are to be developed to account for human performance time in this 3 serial stage situation. Third, the error rate of human cognition system is not quantified in the current model, which is similar to many mathematical model of human performance time that do not consider the error rates [36]. In the next stage of development of the model, a speed & error trade off mechanism is to be incorporated in the model so that it can estimate performance time and error at the same time.

We are integrating mathematical modeling approach with the design of in-vehicle system, providing a special angle to improve the current intelligent transportation systems. Even though the case study and examples covered in this paper are mainly for surface transportation area, the closed-form equations developed in this work might be applied when two serial information processing stages are involved in human-machine interaction.

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Appendix 1. Estimated Value of Parameters

A1(Mesg): 42*3+18*2= 162 ms (Sum of processing time at Server 5, Server6/7, Server 8, Server A/B and Server C in QN-MHP [31])

A2(Radar): 42*3+18*2+SOA+230 ms = SOA+392 ms (230 ms is the average time of an eye movement based on MHP [20])

F1 (Mesg): 18*2=36 ms (two operations in the Server F: a) retrial (stimuli information, 1st or 2nd message) from the working memory server (Server B for textual information) and b) decide which button to press [31])

F2 (Radar): 40*18=720 ms (See Table 2, [31])

C1 (Mesg): 18+24+24=90 ms (Cognitive subnetwork: Server C; motor subnetwork: Servers W and Y [31]).

C2 (Radar): 18+24+24=90 ms [31]

M1(Mesg): 1157+500+352+2*100=2209 ms (finger movement time (0.204*$\log_2$(A/W+1) =0.204$\log_2$(50cm/1cm+1)=1157 ms) [37]; Preset double-click computer time (500 ms, see experimental setup); finger movement time in double clicking (2*176 [38]); and touch screen computer response time (100 ms, see experimental setup).

M2 (Radar): 630+176+100=906 ms (finger movement time (0.204*$\log_2$(A/W+1 [37]) =0.204*$\log_2$(30cm/4cm+1)=630 ms); finger double click time (176, [38]); and touch screen computer response time (100 ms, see experimental setup).

Table 2. Estimation of Number of Mental Operations in the Speeding Judgment Task

	Mental Events/Processes	No. of Operations
1	Retrieve (stimuli of current speed (XX), each digit needs 1 operations) from (working memory server, Server C or B)	2
2	Retrieve (ranges of the speed) from (a long-term memory server, Server H) including: 1) Range 1: upper bound (1), “60” (2); lower bound (1), “55”(2), upper bound includes 60 (2), lower bound includes “55” (2); 2) Range 2: lower bound (1), “61” (2), lower bound includes “61” (2)	15
3	Judge (current speed) is (within range 1 or 2) (2 digits comparison)	2
4	Store (the speed judgment results) into (a working memory server, Server C or B)	1

5	Judge (level of speeding based on speed and distance)
	1) Decide: if (speed is within Range 1) then:
	Retrieve (distance criteria for Range 1) from (Server H) including: lower bound (1), “65” (2), lower bound includes “65” (2);
	Decide: (current distance) is (above the lower bound of the distance criteria or not):
	If (it is above or equal), then (decide to press button “II”) (2-digit comparison)
	Else (press “I” button)
	Store (decision results) into (a working memory server, Server C or B)
	2) Decide: If (speed is within Range 2) then:
	Retrieve (distance criteria for Range 2): including: lower bound (1), “105” (3), lower bound includes “105” (3);
	Decide: (current distance) is (above the lower bound of the distance criteria or not):
	If (it is above or equal), then (decide to press button “II”) (3-digit comparison)
	Else (press “I” button)
	Store (decision results) into (a working memory server, Server C or B)
		1
		5
		2
		1
		7
		3
		1

	Sum	40

Note: All of these processing time refer to young people. For old people, their processing time can be estimated with the method developed by Wu et al. (2007a).